Mathematics-I (Unit-II): Vector Space & Linear Transformations by A.kumar(Ankul kumar)

Most Important Questions for MST-2 (Including 2 Marks & 5 Marks Questions with Problem Statements)

2-MARK QUESTIONS (Short Answer Type)

1. Define a vector space with an example.

2. What is meant by a subspace of a vector space? Give one example.

3. Define linear independence and linear dependence of vectors.

4. What is the basis of a vector space?

5. Define the dimension of a vector space.

6. Write the condition for a set of vectors to form a basis.

7. Define a linear transformation (or linear map).

8. Define the kernel and range of a linear transformation.

9. State the Rank–Nullity theorem (without proof).

10. Define the inverse of a linear transformation.

11. What is meant by the composition of linear maps?

12. Define the matrix associated with a linear transformation.

13. What is meant by the rank and nullity of a matrix or linear transformation?

14. Define an inner product space with an example. (Self-study)

15. State the Gram–Schmidt orthogonalization process. (Self-study)

5-MARK QUESTIONS (Descriptive / Problem-Solving Type)

1. Show that the set of all polynomials of degree ≤ 2 forms a vector space over $\mathbb{R}$. (Verify all vector space axioms.)

2. Find whether the following set is linearly independent: S = {(1, 2, 3), (2, 4, 6), (3, 6, 9)} over $\mathbb{R}^3$.

3. Find a basis and the dimension of the subspace of $\mathbb{R}^3$ spanned by (1, 1, 0), (1, 0, 1), (0, 1, 1).

CU updates

4. Let $V = \mathbb{R}^3$ and define $T: V \to V$ by $T(x, y, z) = (x + y, y + z, z + x)$. Find the matrix associated with T relative to the standard basis.

5. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (x + y, y + z)$. Find the kernel, range, rank, and nullity of T.

6. Verify the Rank–Nullity theorem for the transformation $T(x, y, z) = (x + y, y + z)$.

7. If T_1 and T_2 are two linear maps from $\mathbb{R}^2 \to \mathbb{R}^2$ defined as $T_1(x, y) = (x + y, y)$ and $T_2(x, y) = (x, x - y)$, find the composition $T_2 \circ T_1$ and show whether it is linear.

8. For the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (2x + 3y, x - y)$, find the inverse transformation if it exists.

9. Find the matrix representation of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y + z, 2x + z, y - 2z)$.

10. (Advanced / Self-study) Apply the Gram–Schmidt orthogonalization process to the set $\{(1, 1, 0), (1, 0, 1)\}$ in $\mathbb{R}^3$ to obtain an orthonormal basis.

Tips for MST-2 Preparation

- Practice verifying vector space axioms for given sets.
- Learn how to check linear independence using determinant or row-reduction.
- Be confident in finding basis and dimension of a subspace.
- Focus on computing kernel, range, rank, and nullity of transformations.
- Remember standard matrix representations for linear maps and composition rules.
- For self-study, understand inner product properties and orthogonalization steps.

Cu updates